DAY — **09** SEAT NUMBER | | | | | | |

2025	II	22	1100	**J-316**	(E)

MATHEMATICS & STATISTICS (88)
(COMMERCE)

Time : 3 Hrs. **(15 Pages)** **Max. Marks : 80**

General Instructions :

(i) *All questions are compulsory.*

(ii) *There are six questions divided into two sections.*

(iii) *Write answers of Section-I and Section-II in the same answer book.*

(iv) *Use of logarithmic tables is allowed. Use of calculator is not allowed.*

(v) *For L.P.P. and Time Series graph paper is not necessary. Only rough sketch of graph is expected.*

(vi) *Start answer to each question on a new page.*

(vii) *For each objective type of question (i.e. Q.1 and Q.4) only the first attempt will be considered for evaluation.*

SECTION - I

Q. 1. **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) : (6) **[12]**

(i) If p : He is intelligent

q : He is strong

Then, symbolic form of statement "It is wrong that, he is intelligent or strong" is :

(a) $\sim p \vee \sim q$

(b) $\sim(p \wedge q)$

(c) $\sim(p \vee q)$

(d) $p \vee \sim q$

(ii) $\int \left(x+\frac{1}{x}\right)^3 dx =$

(a) $\frac{1}{4}\left(x+\frac{1}{x}\right)^4 + c$

(b) $\frac{x^4}{4}+\frac{3x^2}{2}+3\log x-\frac{1}{2x^2}+c$

(c) $\frac{x^4}{4}+\frac{3x^2}{2}+3\log x+\frac{1}{x^2}+c$

(d) $\left(x-x^{-1}\right)^3+c$

(iii) $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}dx =$

(a) $\frac{7}{2}$ (b) $\frac{5}{2}$

(c) 7 (d) 2

(iv) The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is

(a) $\frac{32}{3}$ sq. units (b) $\frac{64}{3}$ sq. units

(c) $\frac{16}{3}$ sq. units (d) 64 sq. units

(v) The order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = a^x$$ are ______ respectively.

(a) 1, 1 (b) 1, 2

(c) 2, 2 (d) 2, 1

(vi) The integrating factor of the differential equation

$\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$ is

(a) $\log x$ (b) e^x

(c) $\frac{1}{x}$ (d) x

(B) State whether the following statements are true or false (1 mark each) : (3)

(i) If A is a matrix and K is a constant, then $(KA)^T = KA^T$

(ii) $\int \log x \, dx = x \log x + x + c$

(iii) The differential equation obtained by eliminating arbitrary constants from $bx + ay = ab$ is $\frac{d^2y}{dx^2} = 0$.

(C) Fill in the following blanks (1 mark each) : (3)

(i) The average revenue R_A is 50 and elasticity of demand η is 5, the marginal revenue R_M is _____.

(ii) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx =$ _____ $+ c$

(iii) If $f'(x) = x^2 + 5$ and $f(0) = -1$ then $f(x) =$ _____.

Q. 2. **(A)** Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) Write the converse, inverse and contrapositive of the statement "If a triangle is equilateral then it is equiangular".

(ii) Find x, y, z if $\left\{5\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -2 \\ 1 & 3 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} x-1 \\ y+1 \\ 2z \end{bmatrix}$

(iii) Evaluate : $\int \frac{1}{x(x^6+1)} dx$

(B) Attempt any TWO of the following questions (4 marks each) : (8)

(i) Solve the following equations by the method of inversion :

$2x - y + z = 1$

$x + 2y + 3z = 8$

$3x + y - 4z = 1$

(ii) Find MPC, MPS, APC and APS, if the expenditure E_c of a person with income I is given as :

$E_c = (0.0003) I^2 + (0.075) I$; when $I = 1000$

(iii) Evaluate : $\int_{1}^{2} \frac{dx}{x^2+6x+5}$

Q. 3. **(A)** Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) Find $\frac{dy}{dx}$ if $y=(x)^x+(a)^x$

(ii) Find the area of the region bounded by the parabola $y^2=25x$ and the line $x=5$.

(iii) Find the differential equation by eliminating arbitrary constants from the relation $y=Ae^{3x}+Be^{-3x}$.

(B) Attempt any ONE of the following questions (4 marks each): (4)

(i) Using the truth table, verify $p\vee(q\wedge r)\equiv(p\vee q)\wedge(p\vee r)$

(ii) If $x=\frac{4t}{1+t^2}$, $y=3\left(\frac{1-t^2}{1+t^2}\right)$, then show that $\frac{dy}{dx}=\frac{-9x}{4y}$.

(C) Attempt any ONE of the following questions (Activity) (4 marks each): (4)

(i) Divide the number 84 into two parts such that the product of one part and square of the other is maximum.

Solution :

Let one part be x then the other part will be $84 - x$.

$\therefore \quad f(x) = \boxed{}$

$\therefore \quad f'(x) = 168x - 3x^2$

For extreme values $f'(x) = 0$

$168x - 3x^2 = 0$

$\therefore \quad 3x(56 - x) = 0$

$\therefore \quad x = \boxed{}$ OR $\boxed{}$

$f''(x) = 168 - 6x$

If $x = 0, f''(0) = 168 - 6(0) = 168 > 0$

$\therefore$ function attains minimum at $x = 0$

If $x = 56, f''(56) = \boxed{} < 0$

$\therefore$ function attains maximum at $x = 56$

$\therefore$ Two parts of 84 are $\boxed{}$ and $\boxed{}$

(ii) Solve the following differential equation

$(x^2 - yx^2)dy + (y^2 + xy^2)\,dx = 0$

Solution :

Separating the variables, the given equation can be written as :

$$\boxed{}\,dy + \boxed{}\,dx = 0$$

$$\therefore \quad \left(y^{-2} - \frac{1}{y}\right)dy + \left(x^{-2} + \frac{1}{x}\right)dx = 0$$

$$\boxed{}\,dy - \frac{1}{y}dy + x^{-2}dx + \boxed{}\,dx = 0$$

Integrating we get,

$$\int y^{-2}dy - \int \frac{1}{y}dy + \int x^{-2}dx + \int \frac{1}{x}dx = 0$$

$$\therefore \quad \frac{y^{-1}}{-1} - \square + \frac{x^{-1}}{-1} + \square = c$$

$$-\frac{1}{y} - \frac{1}{x} + \log x - \log y = c$$

$$\log x - \log y = \square + c$$

is the required solution.

SECTION - II

Q. 4. **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) : (6) **[12]**

(i) An agent who gives guarantee to his principal that the party will pay the sale price of goods is called –

(a) Auctioneer

(b) Del credere agent

(c) Factor

(d) Broker

(ii) In an ordinary annuity, payments or receipts occur at

(a) Beginning of each period

(b) End of each period

(c) Mid of each period

(d) Quarterly basis

(iii) Moving averages are useful in identifying

(a) Seasonal component

(b) Irregular component

(c) Trend component

(d) Cyclical component

(iv) If $P_{01}(L)=90$ and $P_{01}(P)=40$, then $P_{01}(D-B)$ is _____.

(a) 65

(b) 50

(c) 25

(d) 130

(v) The objective of an assignment problem is to assign

(a) Number of jobs to equal number of persons at maximum cost

(b) Number of jobs to equal number of persons at minimum cost

(c) Only to maximize the cost

(d) Only to minimize the cost

(vi) The expected value of the sum of two numbers obtained when two fair dice are rolled is _____.

(a) 5

(b) 6

(c) 7

(d) 8

(B) State whether the following statements are true or false (1 mark each) : (3)

(i) If $b_{yx}+b_{xy}=1.30$ and $r=0.75$ then the given data is inconsistent.

(ii) Cyclic variation can occur several times in a year.

(iii) Cost of living index number is used in calculating purchasing power of money.

(C) Fill in the following blanks (1 mark each) : (3)

(i) The amount paid to the holder of the bill after deducting banker's discount is known as _____.

(ii) The simplest method of measuring trend of time series is _____.

(iii) Quantity index number by weighted aggregate method is given by _____.

Q. 5. **(A)** Attempt any TWO of the following questions (3 marks each) : **[14]** (6)

(i) Compute the appropriate regression equation for the following data :

X	1	2	3	4	5
Y	5	7	9	11	13

X is the independent variable and Y is the dependent variable.

(ii) A company makes concrete bricks made up of cement and sand. The weight of a concrete brick has to be at least 5 kg. Cement costs ₹ 20 per kg and sand costs ₹ 6 per kg. Strength consideration dictate that a concrete brick should contain minimum 4 kg of cement and not more than 2 kg of sand. Formulate the L.P.P. for the cost to be minimum.

(iii) Find the mean of number of heads in three tosses of a fair coin.

(B) Attempt any TWO of the following questions (4 marks each) : (8)

(i) Obtain the trend value for the following data using 4-yearly centered moving averages :

Years	**1976**	**1977**	**1978**	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**
Index	0	2	3	3	2	4	5	6	7	10

(ii) Find the sequence that minimizes the total elapsed time to complete the following jobs in the order AB. Find the total elapsed time and idle time for machine B :

Jobs	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**
Machine A	7	16	19	10	14	15	5
Machine B	12	14	14	10	16	5	7

(iii) Five cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability that :

(a) all the five cards are spades

(b) only 3 cards are spades.

Q. 6. **(A)** Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) A house valued at ₹ 8,00,000 is insured at 75% of its value. If the rate of premium is 0.80%, find the premium paid by the owner of the house. If agent's commission is 9% of the premium, find agent's commission.

(ii) Solve the following L.P.P. by graphical method.

Maximize : $z = 4x + 6y$

Subject to : $3x + 2y \leq 12$

$x + y \geq 4$

$x, y \geq 0$

(iii) Defects on plywood sheet occur at random with the average of one defect per 50 sq.ft. Find the probability that such a sheet has :

(a) no defect

(b) at least one defect

(use $e^{-1} = 0.3678$)

(B) Attempt any ONE of the following questions (4 marks each) : (4)

(i) The equations of two regression lines are $10x - 4y = 80$ and $10y - 9x = -40$. Find

(a) $\bar{x}$ and $\bar{y}$

(b) b_{YX} and b_{XY}

(c) r

(d) If var(Y) = 36, obtain var (X).

(ii) Find x if the cost of living index is 150 :

Group	Food	Clothing	Fuel and electricity	House Rent	Miscellaneous
I	180	120	300	100	160
W	4	5	6	x	3

(C) Attempt any ONE of the following questions (Activity) (4 marks each) : (4)

(i) A bill of ₹ 18,000 was discounted for ₹ 17,568 at a bank on 25th October 2017. If the rate of interest was 12% p.a. what is the legal due date?

Solution :

Given $SD = 18{,}000$; $CV = 17{,}568$

$r = 12\%$ p.a.

Now, $BD = \boxed{}$

$= 18{,}000 - 17{,}568$

$=$ ₹ 432

Also $BD = \boxed{}$

$$\therefore \quad 432 = \frac{18{,}000 \times n \times 12}{100}$$

$$n = \frac{432 \times 100}{18{,}000 \times 12}$$

$$n = \frac{1}{5} \text{ years} = \boxed{} \text{ days}$$

The period for which the discount is deducted is 73 days, which is counted from the date of discounting i.e. 25^{th} October 2017 :

October	November	December	January	Total
6	30	31	6	73

Hence legal due date is $\boxed{}$

(ii) Solve the following assignment problem for minimization :

	I	II	III	IV	V
1	18	24	19	20	23
2	19	21	20	18	22
3	22	23	20	21	23
4	20	18	21	19	19
5	18	22	23	22	21

Solution :

Step-I

Subtract the smallest element of each row from every element of that row

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 0 & 6 & 1 & 2 & 4 \\ 1 & 3 & 2 & 0 & 3 \\ 2 & 3 & 0 & 1 & 3 \\ 2 & 0 & 3 & 1 & 1 \\ 0 & 4 & 5 & 4 & 3 \end{bmatrix} \end{array}$$

Step-II

Subtract the smallest element of each column from every element of that column :

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 0 & 6 & 1 & 2 & 4 \\ 1 & 3 & 2 & 0 & 3 \\ 2 & 3 & 0 & 1 & 2 \\ 2 & 0 & 3 & 1 & 0 \\ 0 & 4 & 5 & 4 & 2 \end{bmatrix} \end{array}$$

Step-III

Draw minimum number of lines covering all zeros.

	I	II	III	IV	V
1	0	6	1	2	4
2	1	3	2	0	3
3	2	3	0	1	2
4	2	0	3	1	0
5	0	4	5	4	2

Here minimum number of lines (4) < order of matrix (5)

Step-IV

The smallest uncovered element is 1, which is to be subtracted from all uncovered elements and add it to all elements which lie at the intersection of two lines :

	I	II	III	IV	V
1	0	5	0		3
2	2	3	2	0	3
3	3	3	0		2
4	3	0	3	1	0
5	0	3	4	3	

Step-V

Draw minimum number of lines that are required to cover all zeros :

	I	II	III	IV	V
1	0	5	0	1	3
2	2	3	2	0	3
3	3	3	0	1	2
4	3	0	3	1	0
5	0	3	4	3	1

Here minimum number of lines ≠ order of matrix.

Step-VI

Find the smallest uncovered element (1). Subtract this number from all uncovered elements and add it to all elements which lie at the intersection of two lines :

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 0 & 2 \\ 3 & 3 & 3 & 0 & 3 \\ 3 & \square & 0 & 0 & \square \\ 4 & 0 & 4 & 0 & 0 \\ 0 & 2 & 4 & 2 & 0 \end{bmatrix} \end{array}$$

Now minimum number of lines = order of matrix.

The optimal assignment can be made.

Optimal solution is

$1 \rightarrow \text{I}$

$2 \rightarrow \text{IV}$

$3 \rightarrow \square$

$4 \rightarrow \square$

$5 \rightarrow \text{V}$

Minimum value = $\square$

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